A Code of Practice for the determination of cyclic stress-strain data

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There are no procedural standards for the determination of stress-strain properties where a reversal of stress is involved. The purpose of this Code of Practice is to detail the requirements for cyclic stress-strain (CSS) testing on uniaxial testpieces. CSS testing may entail the use of a single testpiece to produce data over several strain ranges. Alternatively, data from a number of constant strain range tests may be obtained, for example as the by-product of a series of low cycle fatigue (LCF) endurance tests. Procedures for LCF testing are covered by a number of existing Codes of Practice and Standards [1–6], and this document does not recommend any alteration to these. This Code of Practice has been prepared by the CSS Working Party of the ESIS TC11 High Temperature Mechanical Testing Committee.

Historically, CSS results have been reported in terms of a relatively simple power law. However, engineers involved in design and assessment activities are now increasingly tending to use more advanced constitutive relations such as the Chaboche equations [7]. Hence, the model equations available to characterise CSS behaviour cover a range of complexities, with the approach selected being determined by the requirements of the end-user application. These will be influenced by such factors as the type and history of loading, the operating temperature and presence of thermal gradients, the variation of cyclic plastic strain within the component, and the need to determine absolute magnitudes or ranges of stress and strain. The laboratory test procedures defined in this Code of Practice are capable of generating the CSS data required for the full spectrum of model equations currently used in engineering assessment.

In addition to recommending best laboratory practice, this document includes sections on engineering requirements, test data analysis (including the connection between alternative forms of model equation), and the exploitation of existing data. Advice is also given for those circumstances where testpiece material is limited, thus requiring quick methods of data acquisition using block loading techniques. In all cases, the use of cylindrical testpiece gauge lengths is recommended, and only isothermal testing at appropriate temperatures under strain-controlled conditions is covered.

Keywords: cyclic stress-strain data, single step, multiple step, incremental step tests, elastic modulus, cyclic yield stress, hardening, softening, process equations.

1. INTRODUCTION

Cyclic stress-strain (CSS) data are required for engineering application at all stages in the design and assessment of low and high temperature plant, whether in a future life prediction or retrospective analysis. Evaluation of the deformation response of a structure to externally applied loads or constraints is often de-coupled from the calculation of material damage. The response of materials experiencing cyclic strains in service depends on their inherent behaviour and whether such behaviour can be altered by prior loading or after periods of service ageing. This document does not address any form of cracking. Indeed, fatigue crack initiation in the testpiece is to be avoided for valid CSS data collection.

According to engineering requirements (Section 5), CSS data may be gathered to characterise different levels of behaviour, *e.g.* operation below a specified proof strain level, knowledge of stabilised response, or the classification of materials into cyclic hardening, cyclic softening or neutral behaviour (combined with a knowledge of changing cycle-by-cycle response). As computer assessment packages become more sophisticated, the latter type of data is finding increasing use.

The CSS response of materials frequently appears as a by-product of cyclic endurance tests, *e.g.* as properties quoted at the half-life stage. Several Codes of Practice

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and Standards relate to LCF endurance testing [1–6] but these do not specifically state how to measure the CSS response of materials themselves. Such data are often inadequate since it is known that many alloys exhibit cyclic hardening or softening behaviour. In the late 1960s two 'short cut' tests involving block loading were proposed for CSS determination, namely the Incremental-Step [8] and Multiple-Step [9] tests defined below. These have since been used by many investigators. Reviews of typical CSS response using these and other techniques are given elsewhere [10–12].

Conventional LCF tests seek to ascertain the number of cycles to failure (or initiation) at various strain ranges. However, such data alone are generally insufficient to predict the life of a component. A further necessary requirement is a constitutive law that relates stress and strain. This enables the deformation response induced by the external loads to be predicted and hence an estimate obtained for the life of the component. The aim of this document is to highlight the importance of the relationship between stress and strain and to discuss a number of related topics which directly affect CSS testing.

2. SCOPE

This Code of Practice covers the uniaxial testing of nominally homogeneous materials in support of materials research and development programmes, life assessment procedures, design and analysis, validation exercises and other related activities. CSS tests are performed in a similar manner to LCF endurance tests, and hence the same general principles apply (*e.g.* on load train alignment [13,14]). Usually, however, CSS tests are of short duration and are rarely taken to failure. Recommendations are given for the detection of crack initiation, up to which CSS data are considered as valid.

The document defines the acquisition and interpretation of relevant data for engineering requirements. Many of the methods and techniques described here are equally applicable to isotropic and anisotropic materials (*e.g.* single crystals).

The document is restricted to the determination of cyclic plastic (rate-independent) behaviour. Typical strain rates of testing lie in the region 4×10^{-3}/s to 1×10^{-5}/s (1 cycle/min to 1 cycle/h depending on strain range, see Appendix A). However, many of the criteria set for testing and data acquisition are equally applicable when cyclic creep (rate-dependent) behaviour is important.

3. SYMBOLS

A	cyclic strength coefficient in Ramberg–Osgood power law (Eqn (1))
A_m	cyclic strength coefficient in Ramberg–Osgood power law expressed as semi-strain range and semi-stress range (see Eqn (5) and Appendix B)
B_0	constant in sinh form of Ramberg-Osgood law (Eqn (2))
C_1, C_2	constants in Eqns (9,10)
E	elastic modulus
E_{UPLOAD}	elastic modulus determined from tension-going side of CSS loop
$E_{DOWNLOAD}$	elastic modulus determined from compression-going side of CSS loop
k_o	radius of the yield surface (commonly used in analysis of constitutive equations, *e.g.* Appendix C)
N, N_f, N_s	cycle number, cycle number to failure
N_s	cycle number to saturation hardening or softening
P_p	plastic path length (accumulated plastic strain, Eqns (7,8))
R_m	ultimate tensile strength
R_ε	strain ratio ($\varepsilon_{min}/\varepsilon_{max}$)
T	temperature
ΔW_p	plastic work done per cycle (Eqn (6))
Z	reduction of area on tensile overload
β	cyclic hardening exponent in Ramberg–Osgood power law (Eqn (1))
ε, ε_{max}, ε_{min}	strain, maximum strain, minimum strain
$\Delta\varepsilon_e$, $\Delta\varepsilon_p$, $\Delta\varepsilon_t$	elastic strain range, plastic strain range, total strain range
γ_1, γ_2	constants in Eqns (9,10)
σ, σ_{max}, σ_{min}	stress, maximum stress, minimum stress
$\Delta\sigma$	stress range
$\Delta\sigma_o$	constant in sinh form of Ramberg-Osgood law (Eqn (2))
$\Delta\sigma_{y'}$	cyclic yield stress range

NOTE: Stresses and strains are considered in engineering units. The differences in magnitude between true and engineering stresses and strains only become significant above ~3%.

4. DEFINITIONS

For the purpose of this Code of Practice, the following definitions apply:

Elastic modulus. The ratio of stress to strain below the proportional limit of the material.
Stress range. The algebraic difference between the maximum and minimum values of stress, $\Delta\sigma = \sigma_{max} - \sigma_{min}$ (Figure 1).
Total strain range. The algebraic difference between the maximum and minimum values of total strain, $\Delta\varepsilon_t = \varepsilon_{max} - \varepsilon_{min}$ (Figure 1).
Elastic strain range. The stress range divided by the elastic modulus.
Plastic strain range. The difference between total strain range and the elastic strain range, $\Delta\varepsilon_p$ (Figure 1).
Amplitude. Half the range of any of the above variables.
Hysteresis loop. Stress-strain history for one complete loading cycle (Figure 1).
NOTE: Non-stabilised hysteresis loops will not be closed, i.e. during the cyclic hardening/softening stage experienced in certain materials.
Cyclic yield stress range. Stress range required to give plastic strain range of specified proof width, *e.g.* 0.1%, 0.2% proof strain ranges. The chosen value of yield may change during cyclic evolution.
Bauschinger effect. The lowering of the absolute value of the elastic limit in compression following a previous tensile loading and *vice versa*.
Single-step test. As defined in Standard Procedures on LCF [1–6], where a testpiece is taken to a specified number of cycles at a constant range of total strain.
Incremental-step test. A form of block loading where the strain range is incremented every half cycle to a specified

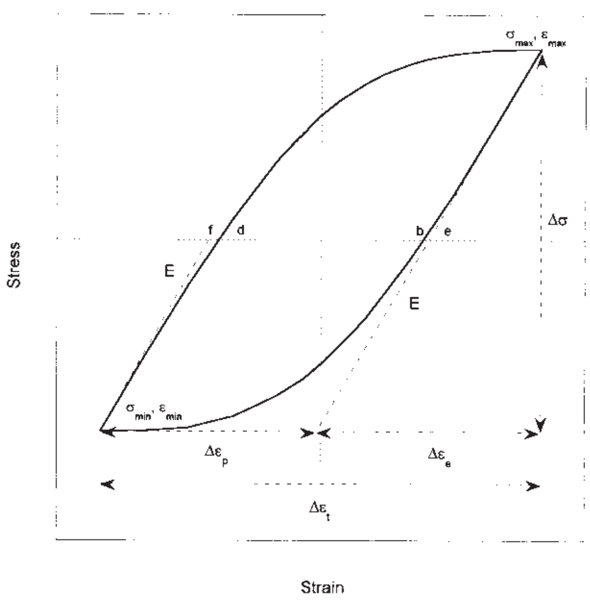

Figure 1 Terms used to characterise cyclic stress strain loop.

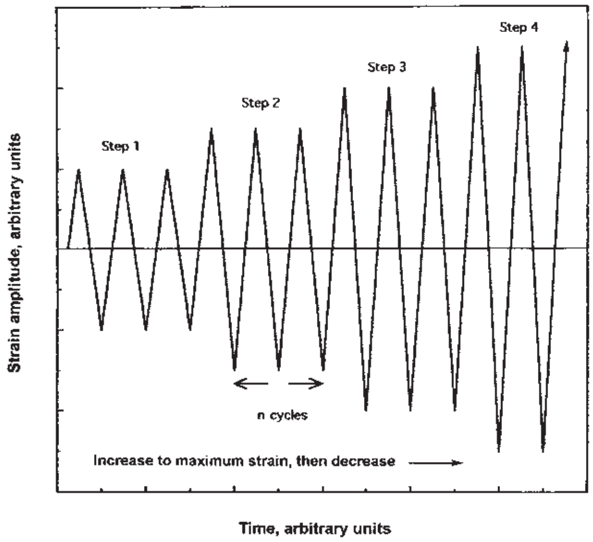

Figure 3 Typical multiple-step test (schematic).

maximum total strain range and then decremented until elastic conditions are again attained. The process is repeated until the locus traced by the tips is reproducible, see Figure 2.

Multiple-Step test. Similar to the Incremental-Step test except that several cycles of equal strain range are specified in each block, as are the number of blocks in the ascending mode to maximum strain (and descending mode, where appropriate to minimum strain range), see Figure 3.

Evolutionary cyclic hardening. In a test conducted at a fixed total strain range, the attendant increase in stress range and consequent decrease in plastic strain range observed cycle-by-cycle, see Figure 4.

Evolutionary cyclic softening. In a test conducted at a fixed total strain range, the attendant decrease in stress range and consequent increase in plastic strain range observed cycle-by-cycle, see Figure 5.

NOTE: Hardening materials usually demonstrate a plateau in a plot of stress range versus cycles, whereas with softening, materials generally appear to continue to soften after an initial rapid period. Some materials may soften after early initial hardening. Materials which

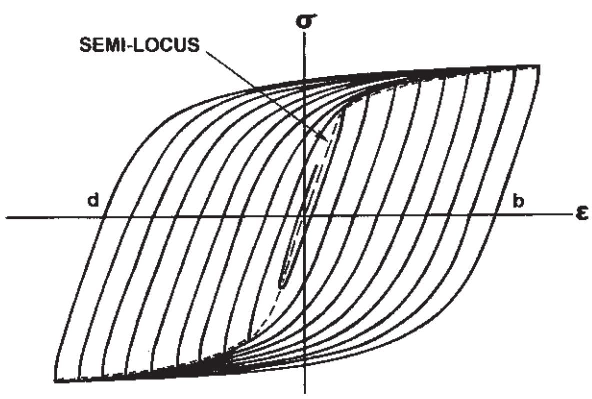

Figure 2 Typical incremental-step test (schematic).

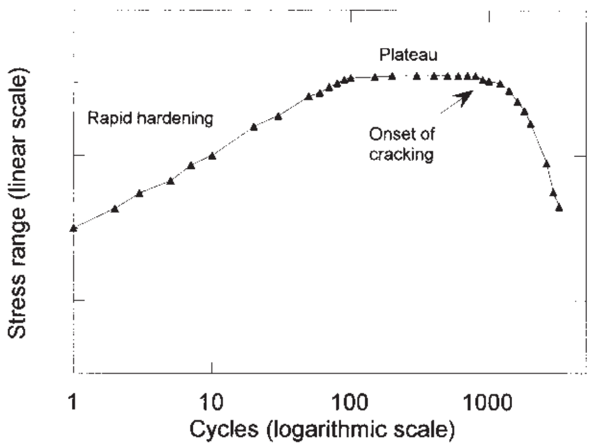

Figure 4 Variation of stress with time for cyclic hardening material.

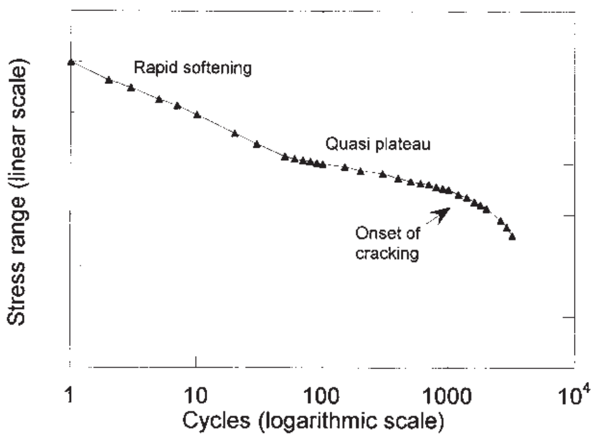

Figure 5 Variation of stress with time for cyclic softening material.

harden or soften slightly may be described as cyclically neutral. These processes also occur during block loading types of test but are more difficult to quantify.

Cyclic pre-conditioning. The amount and nature of any cyclic deformation applied to a testpiece prior to the commencement of a CSS test (evolutionary behaviour). It may be preferred, according to the application, to avoid this step and go straight into the main test.

The material characteristics which may be determined by means of the various tests are compared in Table 1. Further guidance is given in Appendix A.

5. ENGINEERING REQUIREMENTS

The mathematical form of the constitutive laws relating stress and strain is governed by the deformation characteristics of the material, and the envisaged complexity of the structure and the loading to which it is subjected. Answers to the following will influence the selection of constitutive model and assist in the definition of the test matrix.

- Does the material exhibit a non-linear stress-strain response?
- Does the material have a distinct stabilised state or does it continuously harden or soften?
- Is the hardening (or softening) of an isotropic (change in yield stress) or kinematic (change in plastic slope) nature?
- Does the material exhibit a distinct Bauschinger effect [15]?
- Does the material exhibit a relaxation of mean stress under strain control or a strain ratchet under stress control?
- Does the material exhibit any memory of past deformation?

These concepts are further developed in references [7,16–23]. Consideration to constitutive equations is given in Table 2.

To illustrate the need for CSS data of various levels of complexity, a structure subjected to cyclic loading is considered. If the initial stress is greater than the monotonic yield stress, some plastic deformation will occur during the first cycle. For materials, which cyclically harden or soften or are neutral, subsequent deformation may lead to the condition where the local cyclic stress range is less than the saturated cyclic yield stress. Under such conditions, the deformation response may become elastic (shakedown) and assessment calculations are greatly simplified. For cyclically softening materials, the development of this deformation response occurs less often.

If a component is not in a state of shakedown then a constitutive or simple process equation should be used to establish the stress and strain ranges in the region undergoing cyclic plasticity. The material can be conveniently characterised by plotting the stabilised hysteresis loops offset to a common origin, as shown in Figure 6. The CSS behaviour is described by the locus of the tips of these loops, which can be represented by the cyclic form of the Ramberg–Osgood equation [16]:

Table 1 Summary of material characteristics generated by CSS testing procedures

Parameter	Single-Step (¼ Cycle – $N_f/2$) (many testpieces)	Incremental-Step (Single testpiece)	Multiple-Step (Single testpiece)	
			<10 cycles	>100 cycles
Modulus and σ_y for 1st ¼ cycle	Yes	Depending on strain range	Only for lowest $\Delta\varepsilon_t$	Only for lowest $\Delta\varepsilon_t$
Demonstration of hardening or softening.	Yes	Qualitative only	Depending on strain range	Depending on strain range
Stabilised $\Delta\sigma$-$\Delta\varepsilon_t$ properties. Locus or actual shape	Yes	Stabilised properties when no further change in locus	Yes, if hardening or softening effect small	Yes, if hardening or softening effect small
Evolutionary coefficients	Yes	No	Depending on strain range	Depending on strain range
Change/no change in σ_y (isotropic effect)	Yes	Yes	Verification required	Yes
Evolution of plastic slope ($\Delta\varepsilon_p$, ΔW_p)	Yes	Verification required	Verification required	Yes
Detection of crack initiation	Load drop/compliance	Load drop/compliance	Load drop/compliance	Load drop/compliance

Table 2 Capability of various constitutive models to describe particular aspects of cyclic deformation response

Model	Monotonic	Curvature of loops	Bauchinger effect	Cyclic hardening or softening	Ratchet	Memory effect
Bi-linear kinematic (ORNL) [17,18]	Yes	No	Yes	No[¥]	No	No
Bi-linear kinematic with C=f(Wp)	Yes	No	Yes	Yes	No	No
Bi-linear kinematic with C=f(Wp) and memory	Yes	No	Yes	Yes	No	Yes
Mroz [19]	Yes	Yes	Yes	Yes	No	No
Dafalias and Popov [20,21]	Yes	Yes	Yes	Yes	No	No
FRSV [22]	Yes	Yes	Yes	Yes	No	No
Armstrong-Frederick (NL kinematic) [23]	Yes	Yes	Yes	No	Yes	No
Chaboche (NL kinematic + isotropic) [7]	Yes	Yes	Yes	Isotropic or kinematic	Partial*	No
Chaboche (NL kinematic + isotropic + memory) [7]	Yes	Yes	Yes	Isotropic or kinematic	Partial*	Yes

[¥]The ORNL model allows a change between monotonic and cyclic response.
*Depending on number and form of linear kinematic terms.

$$\Delta\varepsilon_t = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{A}\right)^{1/\beta} \quad (1)$$

where A is the cyclic strength coefficient and β is the cyclic hardening exponent. In Eqn (1) and throughout this document, values of A and β are taken to define this full range behaviour, although alternatives are possible (see Section 7.2 and Appendix B). Guidance on the determination of the parameters of such a law are given in Section 7. This simple description of the cyclic response is amenable to simplified design assessments.

In some cases, the conditions which permit the use of simplified methods are not satisfied and the material does not exhibit a stabilised response (*e.g.* when the component, (i) experiences a few major cycles, (ii) comprises isolated regions undergoing cyclic plasticity at differing strain ranges, or (iii) is subject to a particularly complex loading history). In such circumstances, it may be necessary to use the CSS curve corresponding to a particular number of cycles or, more generally, to employ an advanced constitutive model (*e.g.* that of Chaboche [7]) which captures any evolutionary or history-dependent aspects of the material response. Guidance on the constitutive equations available to model more complex CSS situations is given in Table 2 and Appendix C.

Although advanced constitutive laws require a large computational resource, the popularity of these models has increased in step with computer developments. Some commercial finite element codes for structural analysis have implemented standard forms of the models and others permit the models to be programmed in the form of so-called 'user material' sub-routines.

The complexity of these models places additional demands on test data. Sufficient data need to be available to both fit and validate the models. Further recommendations on the types of test to ascertain particular types of behaviour are given in Tables 1 and 2, and Appendix A. Some of the background information on the mathematical formulation of the models is given in Appendices B and C. However, this is of a very limited nature and reference should be made to one of the more substantial texts on constitutive modelling [7,17–23].

In many cases, insufficient data are available to define the CSS characteristics or testing may be unfeasible. In such cases, there is a need to approximate the material response from a minimal amount of information. Because of the frequency with which these problems are encountered, some guidelines are set out in Appendices D and E.

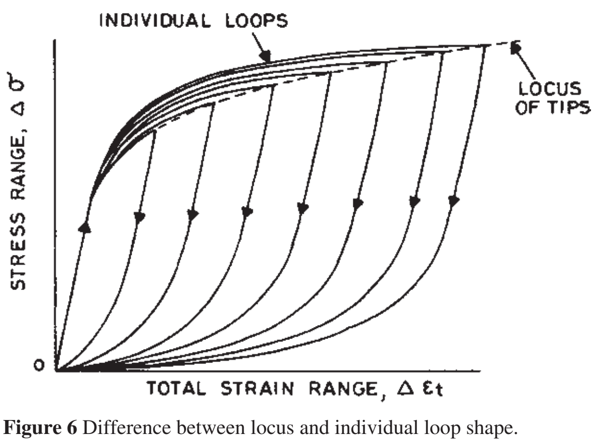

Figure 6 Difference between locus and individual loop shape.

6. MATERIALS TESTING

Where prior knowledge of the behaviour of the class of material under consideration is available, this should be utilised to determine the selection of tests to ascertain the response of the material (see Table 1) and to define the test matrix. In the absence of such information, it is recommended that some simple tests are carried out to identify some key properties identified in Section 5 (*e.g.* does the material ratchet?). This information will also influence the choice of constitutive model.

6.1 Test matrix

The principal variables for a CSS test matrix are total strain range ($\Delta\varepsilon_t$) and temperature (T). Tests are generally performed under strain control because this most closely represents the conditions in many components. The strain ranges and temperatures at which the tests are performed should cover the ranges likely to be experienced within the component. The number of tests performed will be governed by considerations such as the amount of material available, the ranges of strain and temperature, the need to ascertain repeatability, the need to establish batch-to-batch variability.

Ideally, Single-Step tests should be performed at all strain ranges and temperatures in the test matrix to avoid history-dependent effects. However, it is more common to perform a limited number of Single-Step tests augmented by a number of Multiple-Step tests at intermediate strain ranges and temperatures. If history-dependent effects are negligible then, preferably, Multiple-Step or, possibly, Incremental-Step tests may be used to gather data at intermediate temperatures and strain ranges. If history dependence is not negligible then, depending upon the final application, a full material characterisation may be necessary.

Secondary variables that require some consideration are strain ratio and strain rate. It is common practice in LCF endurance testing to use a fully reversed strain range ($R_\varepsilon = -1$). This practice is, however, not universal because few components actually operate under fully reversed conditions. Under mechanical loading a significant mean strain (and hence mean stress) can arise and such conditions are represented more accurately by larger strain ratios ($R_\varepsilon > 0$). Although the influence of strain ratio on the CSS range is often small, it is good practice to include a few tests at different strain ratios.

Although the models considered here do not address strain rate effects, care should be taken in the selection of the strain rate for high temperature tests within the creep regime of the material. Care should also be taken if the material exhibits rate dependence at lower temperatures (*i.e.* due to cold creep).

6.2 Testing procedure

The information which may be determined from the testing procedures described below is summarised in Table 1.

6.2.1 Apparatus

The testing machine calibration, longitudinal and lateral stiffness, alignment and temperature control requirements should be taken as those recommended in standard procedures for high temperature LCF endurance testing [1–6]. A reverse-stress servo-controlled testing machine is required with minimum backlash, and high lateral stiffness (*e.g.* 3 kN/mm).

In high temperature tests, any form of testpiece heating giving the necessary degree of temperature control may be used, *i.e.* conventional furnace, RF heating, resistance heating, radiant heating *etc*. For heating forms other than conventional, careful attention should be paid to thermocouple attachment. Standard beaded thermocouples are not acceptable since they can heat up independently of the testpiece gauge section. Spot welding near gauge length extremities is recommended, appropriate calibrations being carried out with a thermocouple spot welded at the gauge centre on a dummy testpiece. *(NOTE: Spot welding at the gauge centre of testpieces is known to cause premature crack initiation in some materials [24] which could invalidate further CSS testing.)*

Strain should be measured using a high sensitivity extensometer on a well defined gauge length on a parallel cylindrical testpiece. The use of side-contacting gauge length extensometers is recommended, although extensometers mounted on knife edges or ridges within the testpiece gauge length are acceptable. Displacements measured outside the gauge length require a shoulder correction factor and their use is discouraged.

It is recommended that stress-strain data are captured with a computerised data acquisition system (see Section 6.3.1). As a minimum, an *X–Y* recorder (for the load-gauge length displacement signals, which provide stress-strain hysteresis loops) and a strip chart (*X–t*) recorder (which provides a history of ± stress amplitude with cycles) should be provided.

6.2.2 Testpieces

Recommendations for testpiece gauge length, fillet radii and diameter are given elsewhere [1–6]. Similarly, testpiece preparation procedures shall be those advised in these standard references for high temperature LCF endurance testing. In practice, the surface finish tolerances may be less important for CSS testing when crack initiation is not under investigation. However, in cases where several strain range steps are sought from a single testpiece, the best possible finish is recommended to avoid premature fatigue crack initiation.

6.2.3 Testing considerations

This section assumes a symmetrical cycle with respect to strain ($R_\varepsilon = -1$). However, tests may be carried out at positive or negative values of R_ε according to the application. In such cases it will be necessary to report the behaviour of corresponding tensile and compressive peak stresses, since in some materials a significant mean stress is sustained. Triangular cycling is recommended (see Appendix A), since sinusoidal cycling can give rise to a 'rounding' of hysteresis loops at the tips.

6.2.3.1 Single-Step tests

When CSS data result from LCF tests, the tests should be carried out in accordance with the guidance given in the standard procedures [1–6]. Conventionally, the following data are reported at half life:

- total strain range (constant)
- plastic strain range
- stress range (tension and compression components specified)

It is clear that 'half life' depends on the criterion adopted for 'failure' [25]. However, for some applications, evolution of CSS behaviour from monotonic deformation is more important. In such circumstances, cyclic deformation at discrete intervals may be required as being more representative of service situations (*e.g.* 1st and 10th cycle behaviour), see Appendices D and E.

6.2.3.2 Incremental-Step test

The Incremental-Step test was introduced by Landgraf *et al*. [9]. The total strain limits on a single testpiece are gradually and symmetrically increased after each half-cycle to a decided maximum with the *X–Y* plotter (or data acquisition system) continuously in operation. The superposition of the loops then gives a clearly defined locus of tips as shown in Figure 2. The increments are then gradually decreased until the starting point is reached and the whole process repeated until the locus is reproducible (for saturation). The material is then in the steady state and characterising parameters are obtained from this final locus.

6.2.3.3 Multiple-Step tests

In a Multiple-Step test, a sample is loaded at a fixed condition (normally $\Delta\varepsilon_t$) for a predetermined number of cycles or until saturated conditions are obtained. The test conditions are sequentially modified until sufficient blocks of loading have been determined to characterise the material.

Starting from a specific strain range, the test requires periodic incrementing of the control variable (*i.e.* after a chosen number of cycles), in most cases strain amplitude, in the ascending mode. At maximum strain amplitude the sequence is repeated in the descending mode (Figure 3). The test is generally performed under computer control, but in some circumstances it may be done manually. Where final amplitudes are high (and hence corresponding fatigue endurances low) it should be demonstrated that the properties have not been affected by cracking. This is particularly important when repeat tests have been performed on the same testpiece.

It is recommended that data for different temperatures be obtained on individual testpieces to avoid unknown history-dependent effects, unless this can be demonstrated not to be important, or if the material available for testing is limited.

A typical test would require the following input parameters:

- minimum strain amplitude
- maximum strain amplitude
- number of steps between minimum and maximum values
- number of cycles per step level

A typical specification for such a test, with a view to determining sufficient information to select the form of constitutive model and determine some basic parameters, would be: 3 to 5 blocks of 10 to 50 cycles with increasing strain ranges, followed by 2 blocks of 10 to 50 cycles with decreasing strain ranges (coinciding with the $\Delta\varepsilon_t$ used for the increasing blocks).

6.2.3.4 Stopping the test

It is emphasised that significant loss in area due to cracking in the gauge section cannot be allowed, for the CSS data to be valid. Typical failure criteria in a conventional LCF test are: (i) specified percentage decrease of peak tensile load; (ii) specified compliance change (ratio of apparent elastic moduli in tension and compression; (iii) specified percentage decrease in maximum tensile stress to the maximum compressive stress, and (iv) appearance of a 'cusp' in the hysteresis loop [1–6,26]. It is noted that some of these criteria are relatively coarse so there may well be a case for rejecting CSS data immediately preceding them. At one extreme the cusp method corresponds approximately to a 15% loss in area due to cracking while at the other a 2% drop in tensile load corresponds to the limit of detection i.e. 2% loss in area [26]. For testpieces whose gauge length is approximately twice the diameter the loss in area is directly proportional to the load drop or compliance ratio [26]. *(NOTES: In Single-Step tests, cyclic hardening or softening can sometimes overshadow condition (i). Condition (i) is also difficult to apply in Incremental-Step or Multiple-Step tests since there is no reference point.)*

6.3 Data acquisition

It is important that the CSS test is performed with an accurate control system. Although poor control appears not to affect overall loop shape it becomes significant when determining the parameters for advanced constitutive models. A particular concern is fidelity of the tips of the loops which are often rounded due to poor control or, at high temperature, to creep deformation. This can complicate the definition of the elastic loading/unloading portion and hence the definition of elastic modulus and cyclic yield stress. Inaccuracies in elastic modulus affect the plastic strain range and hence the accumulated plastic work.

6.3.1 Data distribution

Sufficient (σ,ε) data points should be recorded to provide an adequate description of the loop shape. In general, no less than 200 points should be recorded, while more than 500 points are unnecessary.

Ideally, individual (σ,ε) data points should be equi-spaced about the hysteresis loop. This requires the data acquisition system to be able to recognise changes in both stress and strain.

When a material is hardening or softening, the hysteresis loop will not be closed and some judgement may be necessary.

6.3.2 Single-Step tests

The first ¼ cycle (initial loading portion) should always be recorded and compared with the monotonic stress-strain curve. It is also good practice, particularly if a detailed constitutive model is to be developed, to record each of the first 10 hysteresis loops because the majority of the evolutionary behaviour is often contained in these cycles. Most evolutionary behaviour is related to the accumulated plastic work and therefore it is adequate to record hysteresis loops in logarithmic intervals, *e.g.* 10, 30, 100, 300, 1000, 3000 …. Recording every loop is unnecessary. However, hysteresis loops should be gathered at close enough intervals (*e.g.* for each 2% change in stress range) to characterise the whole of the test.

6.3.3 Multiple-Step and Incremental-Step tests

For Multiple-Step and Incremental-Step tests it is essential that every hysteresis loop is recorded, especially the transitions from one loop to the next during an increase, or decrease in strain range.

7. TEST DATA ANALYSIS

Analyses may be performed separately at the completion of testing, or continuously during the test by the use of suitable computer software. However, this Code of Practice does not preclude the use of other analytical techniques where these are deemed to be more accurate or more appropriate. The analysis of CSS data to determine elastic modulus, and the parameters of process equations (*e.g.* Ramberg-Osgood) and constitutive equations (*e.g.* Chaboche) is considered in the following sub-sections.

7.1 Elastic modulus

The elastic modulus is an extremely important parameter since it ultimately defines the elastic strain range and consequently parameters such as the plastic strain range and the cyclic yield stress. Consequently, the correct estimation of this parameter from test data is essential for accurate representation of the material response. The elastic modulus (E) may vary during a test. However, in many instances, the variation is not significant.

An initial measurement of E can be obtained by cycling the testpiece at room temperature at a stress or strain level below the elastic limit. Thomas *et al.* [1] recommend loading up to not more than 50% of the 0.1% proof stress. For practical purposes, this value of E is used to check that the test equipment is working correctly. If the measured value of E deviates more than 5% from the expected value, the force, strain and temperature measuring equipment should be reset and the test restarted [6].

The elastic modulus can be determined from cyclic data as follows:

- At the start of the test E can be estimated by cycling within the elastic limit of the first quarter of the hysteresis loop at the test temperature for each test testpiece [1]. If the values are unacceptable the methodology outlined for the initial measurement of E should be followed.
- Alternatively, for each hysteresis loop recorded, calculate E from the average of the tension-going (E_{UPLOAD}) and compression-going ($E_{DOWNLOAD}$) sides of the loop as there is often a slight difference between E_{UPLOAD} and $E_{DOWNLOAD}$. The reported elastic modulus for each testpiece is the average of this value over the number of cycles recorded. *(NOTE: Sufficient numbers of stress/strain points need to be recorded within the elastic region of the hysteresis loop to accurately estimate the elastic modulus (E).)*

Some differences may be observed between E, E_{UPLOAD} and $E_{DOWNLOAD}$. These should not be excessive (<10%). A sudden marked difference between E_{UPLOAD} and $E_{DOWNLOAD}$ can indicate cracking in the testpiece thereby making the loops invalid for the determination of CSS data. In some instances, variations in modulus are likely to be due to fitting errors, particularly if the elastic portion of the loops is small. In such cases it might be more appropriate to use a single value of modulus determined from a tensile test or from the first ¼ cycle of the cyclic test.

For the estimation of elastic modulus from CSS data, two calculation methods are recommended; each has its own advantages depending on the shape and quality of the hysteresis loop. It should be emphasised that the elastic modulus determined from tensile data or dynamic tests may differ substantially from the values obtained from loops.

(i) In the first method, illustrated in Figure 7a, a least squares linear fit is formed over a defined stress range measured from some offset strain at the loop tips. This method is relatively simple to use but the calculated value of modulus can be sensitive to the choice of stress range and offset, particularly if the elastic range is small.
(ii) In the second method, illustrated in Figure 7b, a moving least squares linear fit, with error bands, is attempted over a portion of the hysteresis loop with the fit progressively moving from the plastic portion of the loop to the elastic portion of the loop. The elastic modulus is obtained from the region over which the longest straight line can be fitted for the given error bands. This method is more general than the first but somewhat more difficult to implement. The error band depends on the quality of the test data (uncertainty associated with individual (σ,ε) points) but 0.005% strain is typically suitable. It may sometimes be necessary to identify and remove rogue points before fitting.

7.2 Fitting process equations

Process equations enable data from a set of tests to be correlated. Common examples for CSS data are the power-law (Eqn (1)) and sinh (Eqn (2)) forms of the Ramberg–Osgood equation:

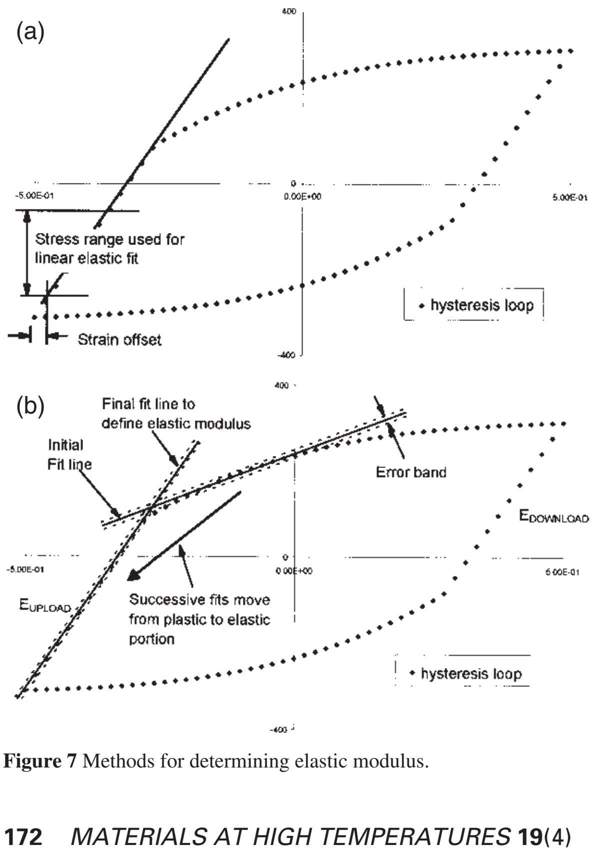

Figure 7 Methods for determining elastic modulus.

$$\Delta\varepsilon_t = \frac{\Delta\sigma}{E} + B_0 \sinh\left(\frac{\Delta\sigma}{\Delta\sigma_o}\right) \tag{2}$$

Although such equations are usually used for describing the shape of the CSS curve at stabilised conditions, it is possible to fit a family of curves, one for each of a given number of cycles, to describe evolutionary behaviour. Process equations are useful for simplified design and assessment methods but they should be distinguished from constitutive equations which define the strain response in conditions of (in principle) arbitrary variations of stress and of temperature with respect to time. Moreover, process equations may sometimes provide a convenient and useful summary of data for later interpretation within constitutive equations.

For Single-Step and Multiple-Step tests, the reported data define the saturated cyclic stress and strain ranges. Such data are widely available in the published literature; being the result of investigations into cyclic deformation behaviour or incidentally determined in LCF endurance tests (see, for example, the compilations of data by Boller and Seeger [28]).

CSS properties are, almost invariably, determined under strain control; *i.e.* a sample is cycled between total strain limits and the resulting cyclic stress range is measured. Thus, the total strain range is the explanatory or independent variable and the stress range is the response or dependent variable. A practical difficulty is that the Ramberg–Osgood power law, Eqn (1), cannot be rearranged to allow the stress to be expressed as a function of total strain. However, the two terms on the right hand side of Eqn (1) imply that the total strain range can be partitioned into elastic and plastic strain ranges. Hence:

$$\Delta\varepsilon_p = \Delta\varepsilon_t - \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{A}\right)^{1/\beta} \tag{3}$$

and

$$\Delta\sigma = A(\Delta\varepsilon_p)^{\beta} \tag{4}$$

Although the plastic strain range cannot be taken to be a truly independent variable, it is treated as such here.

Best estimates of A and β can be obtained by a nonlinear regression analysis or by a linear regression analysis of the log-transformation of Eqn (4). The former method is preferred. The results of this type of analysis indicate that the 95% confidence limits on the predicted values, based on uncertainties in both A and β, diverge widely from the mean line. In some cases, the divergence is so wide as to be of little practical use. However, consideration of Eqn (3) shows that β determines the hardening rate, and experience suggests that whilst this may be a material characteristic it does not vary significantly between casts of the same material. In contrast, A is related to the yield strength which, on the basis of tensile data, is known to vary significantly from cast to cast. The nonlinear regression procedure can be implemented to reflect this behaviour. The best estimates for the two parameters are determined allowing both A and β to vary. The procedure is then repeated for the upper and lower 95% confidence predicted values which are fitted to Eqn (4), while retaining the best estimate of β derived from the data, to provide a functional relationship for the bounding values. A typical set of data together with the mean, upper and lower 95% confidence limits of the Ramberg–Osgood relationship is shown in Figure 8.

Some investigators prefer to work in terms of semi ranges of stress and strain in Eqn (1) [12]. In this case, and provided the tension and compression stress amplitudes are equal, the term A is replaced by A_m where:

$$A = A_m \cdot (2)^{1-\beta} \qquad (5)$$

The term β is unaffected (see also Appendix B). This form of the Ramberg–Osgood CSS equation is often used for comparison with monotonic tensile behaviour.

In general, plotting locus data is regarded as a reasonable approximation to the path traced at any given amplitude, see Figures 2 and 6. However, it is permissible to curve fit the actual *shape* (*i.e.* path traced) from any individual hysteresis loop in the tension-going or compression-going direction by regression analysis, provided the corresponding total strain range is quoted. This may be more accurate when determining intermediate values (see Appendix B).

The S-like curve provided by the Incremental-Step test, Figure 2, demonstrates the tensile and compressive components of the locus. Each half may be curve fitted to provide a locus value for A_m.

7.3 Fitting constitutive equations

Constitutive equations to describe the rate-independent cyclic response of metals can be divided into two categories: (i) simplified equations which describe the stabilised response; and (ii) fuller equations which also describe the evolutionary response. To determine the parameters of either category, it is necessary to compute a number of summary quantities from the hysteresis loops which are then related to various parameters of the constitutive equation. For both categories of constitutive equation it is necessary to determine the elastic modulus, the stress range, the cyclic yield stress, the loop area and the plastic strain range. For the evolutionary models it is necessary to describe the evolution of stress range (under total strain control) in terms of some evolutionary variable such as the accumulated plastic work or the plastic path length. In this section guidance is provided on the determination of these various parameters and their subsequent use to determine the coefficients of constitutive equations. The methods discussed are equally applicable to hysteresis

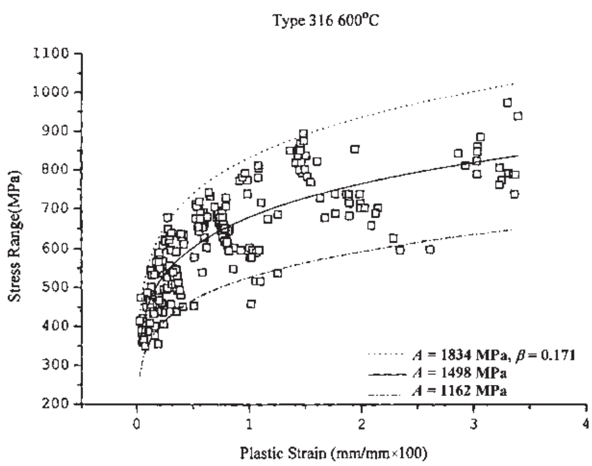

Figure 8 Analysis of typical CSS data from many sources.

loops from any type of test (stress control, multiple-step or incremental-step).

The methods described here are discussed in the context of a stable, closed, hysteresis loop. During the first few cycles of a test, or over the entire test for some materials that harden or soften, each hysteresis loop is not strictly closed. In such cases it is permissible to work off each branch of the loop (loading or unloading) for the determination of elastic modulus and yield stress and to effectively ignore the lack of closure when determining the loop area. The errors associated with this are usually negligible. This approach also enables the quantities to be determined from multiple or incremental-step tests. If the lack of closure is significant this should be reported. *(NOTE: When calculating cumulative quantities such as plastic work or path length one should not neglect the first ¼ cycle (first loading). This contribution can be significant, in particular at large strain ranges where few cycles are required to cause failure, especially when the test is at positive mean strain.)*

7.3.1 Cyclic yield stress

Having defined the elastic modulus, it is then straightforward to determine a value for the cyclic yield stress. Common definitions are based on the limit of proportionality or some offset plastic strain. The former of these can be subjective since few metals exhibit a distinct yield point under cyclic conditions. If an offset plastic strain (*e.g.* (0.01%) is used then the value should be small and will often depend upon the quality of the loop. If method (ii) has been used to determine the elastic modulus (Section 7.1, Figure 7b), then the yield stress can be defined by the point where the hysteresis loop falls outside of the error band of the linear fit.

The evolution of the yield stress with cycles will enable the hardening or softening characteristics of the material to be defined as isotropic or kinematic or some combination of the two (Appendix C). Changes in yield stress can also be traced via the variation of the values A and β during evolution, see Appendix E.

7.3.2 Plastic strain range

Having ascertained the elastic modulus the plastic strain range is given by Eqn (3) and corresponds to *ef* in Figure 1. If, under constant total strain range, the value of $\Delta\varepsilon_p$ reduces cycle by cycle the material is classed as cyclic hardening. If the plastic strain range increases cycle by cycle the material is classed as cyclic softening. Note that the commonly adopted definition of plastic strain range as the loop width on the strain axis, *e.g. db* in Figures 1 and 2 (or the maximum width) [2,6], is only an approximation and may be a severe underestimate if the yield stress is small (see Appendices B and C).

7.3.3 Plastic work (loop area)

The cumulative plastic work, obtained by summing the plastic work for each cycle, often controls the evolution of cyclic hardening or softening and can also be used as a failure criterion. The plastic work in a cycle corresponds to the area of the hysteresis loop which, for a closed loop, is given by:

$$\Delta W_p = \oint \sigma d\varepsilon_p \qquad (6)$$

This integral can be calculated for (σ,ε) pairs using the trapezium rule. The cumulative plastic work is obtained by summing the contribution from each loop. It is common to assume that little change occurs on a cycle-by-cycle basis between successive hysteresis loops for which data are stored, and hence it is unnecessary to store every loop. The fractional change of loop area per cycle is normally much less than that of the plastic strain range and the assumption is usually adequate.

7.3.4 Plastic path length

The plastic path length is a measure of the accumulated plastic strain (independent of sign). It is obtained by summing the plastic strain contribution for each cycle. Again, this parameter can assist in correlating both deformation and failure. It is defined as follows:

$$P_{\mathrm{p}} = \int d\varepsilon_{\mathrm{p}} = \int \left(d\varepsilon - \frac{d\sigma}{E} \right) \quad (7)$$

As for plastic work, this integral can easily be computed using the trapezium rule on successive (σ,ε) pairs. Alternatively, it may be evaluated in terms of plastic strain ranges as

$$P_{\mathrm{p}} = 2 \sum_{1}^{N} \Delta\varepsilon_{\mathrm{p}} \quad (8)$$

to which the plastic strain contribution from the first quarter cycle should be added.

7.3.5 Determination of constitutive coefficients

There are many constitutive equations available to describe the cyclic plastic response of metals, some of which are outlined in Appendix C. Practical equations can usually be manipulated such that the hysteresis loop characteristics can be expressed in terms of the coefficients of the model. The references describing such models are generally accompanied by an account of the data fitting procedure for each model (*e.g.* [7,20,21]). In principle, the number of material parameters should be as few as possible. As an example, a procedure is outlined here for the Chaboche model with two kinematic hardening variables, which is usually sufficient to capture the Bauschinger effect.

The Chaboche model with kinematic hardening, but without functions of cumulative plastic strain, is simple to fit because it only describes the stabilised hysteresis loops. The basic approach is to manipulate the constitutive model such that the stress range can be expressed as a function of the plastic strain range. The constitutive coefficients can then be computed using some numerical optimisation scheme which minimises the difference between the experimental and the predicted CSS curves. The CSS curve can be expressed as:

$$\Delta\sigma = \frac{2C_1}{\gamma_1} \tanh\left(\frac{\gamma_1}{2} \Delta\varepsilon_{\mathrm{p}} \right) + \frac{2C_2}{\gamma_2} \tanh\left(\frac{\gamma_1}{2} \Delta\varepsilon_{\mathrm{p}} \right) + 2k_0 \quad (9)$$

from which the constitutive parameters C_1, γ_1, C_2, γ_2 and k_0 can be determined. The influence of the constitutive parameters on the shape of the CSS curve is illustrated in Figure 9. It is generally also necessary to ensure that the model provides a reasonable description of the monotonic stress-strain curve. The equation for the monotonic stress-strain curve can be derived in a similar manner

$$\sigma = \frac{C_1}{\gamma_1}\left(1 - \exp(-\gamma_1 \varepsilon_{\mathrm{p}})\right) + \frac{C_2}{\gamma_2}\left(1 - \exp(-\gamma_2 \varepsilon_{\mathrm{p}})\right) + k_0 \quad (10)$$

and the quality of the predictions checked against the test data.

In general, it is efficient to make an informed decision about the maximum strain range of interest to avoid the introduction of unnecessary complexity associated with fitting both large and small strain ranges.

If the aim is to describe the evolutionary CSS response then a similar approach can be adopted but, taking the Chaboche equations outlined above as an example, it is now necessary to describe the evolution of the parameters C_1, γ_1, C_2, γ_2, and k_0 as functions of the cumulative plastic work or plastic path length. The details of this are beyond the scope of this document and reference should be made to one of the texts on constitutive modelling [7,20,21].

8. REPORTING

All information regarding tested material, machine control parameters, temperature, strain rate *etc.* shall be reported as for high temperature LCF endurance testing [1–6]. Those parameters specific to single testpiece CSS testing shall also be recorded, including any cyclic preconditioning.

The total strain range, plastic strain range and stress range data at each point used in regression analyses should be tabulated, together with sufficient information to identify from which cycle within each strain level block the data were generated. In block loading type tests it should be recorded whether the data came from an ascending or descending strain step period, or whether an average curve was fitted to both the ascending and descending data. This is important for example in single crystals, or polycrystalline materials with limited slip systems which may experience ‘jerky flow’ in the ascending mode, but behave conventionally in the descending mode. It should also be recorded whether the data came from the initial loading sequence, or for example from a subsequent repeated loading sequence. (*NOTE: The stress range should always be reported in terms of tensile and com-*

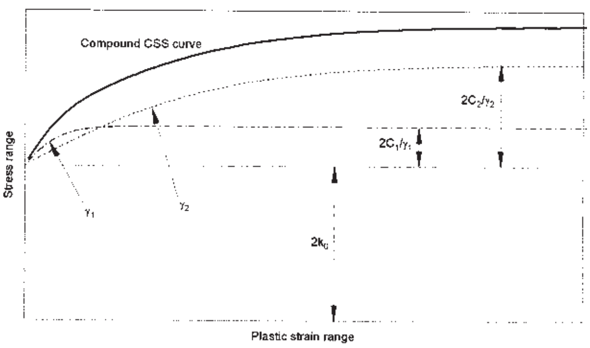

Figure 9 Illustration of the constitutive parameters on the shape of the CSS curve for a Chaboche type equation with two kinematic hardening variables.

pressive components, even though they may not necessarily be utilised in the curve fitting procedure. Some materials, notably single crystals, can show marked asymmetry in stress, even for tests at $R_\varepsilon = -1$.)

The coefficients used to describe the selected process or constitutive model should be clearly tabulated, with associated statistical error estimates.

9. UNCERTAINTIES

Uncertainties in CSS data and the derived process/constitutive equation parameters may arise from a number of sources associated with material variability, testing practice and the analytical procedures employed. The determination of such uncertainties in accordance with ref. [29] is not considered in the present document. However, guidance on the determination of uncertainties in LCF testing is given in ref. [30].

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Appendix A
Tests to characterise aspects of cyclic deformation response

To develop constitutive equations to describe the CSS behaviour it is necessary to ascertain key aspects of the material response, such as cyclic hardening or softening. The table below provides guidance on which tests should be performed to elucidate particular aspects of the response.

Feature of deformation response	Test
Monotonic stress-strain curve	Tensile test or first ¼ cycle of CSS test
Curvature of hysteresis loops	Any cyclic test; but the relative importance of the curvature is ascertained from the CSS curve
Bauschinger effect	Any test with a stress reversal that results in reverse plasticity
Isotropic hardening	Any cyclic test with many hysteresis loops (preferably several orders of magnitude in cycles) and, preferably, various strain ranges
Kinematic hardening	As for isotropic hardening
Ratchetting / mean stress relaxation	Cyclic tests at fixed strain range with a non-zero mean strain (R_ε< >-1); or tests at fixed stress range with non-zero mean stress (R_σ< >-1)
Memory effects	Step-up and step-down multiple step or incremental step tests

For cycles with a triangular waveshape, the relation between (reversed) strain rate, $\dot{\varepsilon}$, and frequency, ν, is given by:

$$\nu = \frac{\dot{\varepsilon}}{2\Delta\varepsilon_t} \tag{A1}$$

Appendix B
Stress/strain origin for process equations

The value of the parameter A in Eqns (1,4) (main text) changes its value according to the origin of stress and strain chosen. **It is therefore essential that this is taken into account according to the type of analysis undertaken**.

B.1 ORIGIN AT PEAK TENSION OR COMPRESSION STRESS (‘RELATIVE COORDINATES’)

In this case, all hysteresis loops providing the data are superimposed with their tips at one origin, 0, shown for the instance of peak compression in Figure 6. The locus of tips is then curve fitted as discussed in Section 7.2. The convention in this Code of Practice is that the values of A and β are defined in this way. (*NOTES: If required for a more accurate assessment, the actual shape of any tension-going or compression-going curve may also be curve fitted. The locus depiction is regarded as an approximation describing average behaviour over a range of strains. If the path taken and locus points actually coincide, the material is said to exhibit Masing behaviour [31].)*

B.2 ORIGIN AT BEGINNING OF FIRST (¼) CYCLE

Many investigators describe their results in terms of semi-range stress and strain so that the locus form of Eqn (1) (main text) for example becomes:

$$\frac{\Delta\varepsilon_t}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2A_m}\right)^{1/\beta} \tag{B1}$$

This form is often used to compare monotonic with CSS behaviour, hence A has been given the suffix ‘m’. Similarly, Eqn (4) (main text) becomes:

$$\frac{\Delta\sigma}{2} = A_m\left(\frac{\Delta\varepsilon_p}{2}\right)^{\beta} \tag{B2}$$

Referring to Figure 2, Eqn (B1) describes either the tension or compression arm of the S-like locus, with the origin at the original starting point. On this basis therefore, comparison may be made with the monotonic curve.

It may be shown [12,31] that:

$$A = A_m \cdot (2)^{1-\beta} \tag{B3}$$

see Eqn (5).

B.3 ORIGIN AT ZERO STRESS CROSSING POINT

Referring to Figure 2, some analytical routines (see for example [32]) prefer to set up a new origin at the zero stress crossing points, shown as d and b in Figure 2.

Total strain, ε_t*, measured from these points (denoted by an asterisk) is thus approximated by:

$$\epsilon_t^* = \frac{\sigma}{E} + \left(\frac{2 \cdot \sigma}{A}\right)^{1/\beta} \tag{B4}$$

where it is assumed that $2\sigma = \Delta\sigma$ Eqn (B4) thus takes the full range of plasticity but only half the elastic range and on this basis also, comparison may be made with the monotonic curve.
An example of Eqn (B4) is provided in Figure B1. It is seen that at all stages it overestimates the tensile stress. It may be shown that a better approximation for Eqn. B4 is:

$$\epsilon_t^* = \frac{\sigma}{E} + \left(\frac{2\sigma'}{A}\right)^{1/\beta} \tag{B5}$$

where

$$\sigma' = \sqrt{\sigma^{1-\beta}\sigma_{max}^{1-\beta}}$$

It is seen that a much better fit is obtained to the original curve in Figure B1. However, the disadvantage is that the maximum semi-stress, σ_{max}, must be inserted for each particular case.

B.4 DEFINITION OF PLASTIC STRAIN RANGE AND SOURCE OF DISCREPANCIES

Published LCF Standards are not in agreement on the method of measuring plastic strain range, $\Delta\varepsilon_p$. References [2] and [6] determine the width of the hysteresis loop at zero (strictly mean) stress while Refs [3] and [5] take the difference between the total and elastic strain range:

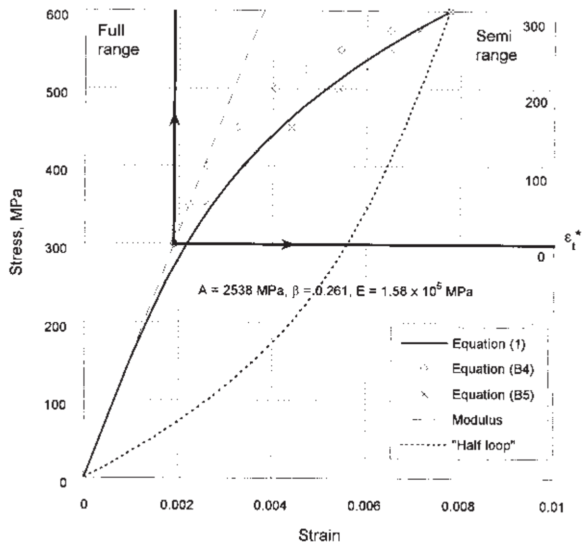

Figure B1 Comparison of CSS equation fits.

$$\Delta\varepsilon_p = \Delta\varepsilon_t - \frac{\Delta\sigma}{E} \tag{B6}$$

See also Eqn (3). In Figure 1, the first definition is shown as *db* whereas that due to Eqn. B6 is shown as *fe*.

These definitions are not inconsistent if it is accepted that the non-linear unloading is due to the Bauschinger effect whereby, owing to the presence of back stresses (which change their sign during the execution of a hysteresis loop), the tension- and compression-going cyclic yield points appear to be permanently depressed compared with the monotonic value [15]. The Masing theory is also able to predict the curvature on unloading [31]. In relative co-ordinates, as defined in Section B1, the power law Eqns (1) and (4), for example, are able to predict the quantities *fd* or *be* shown in Figure 1 by substituting the value $\sigma = \Delta\sigma/2$.

Appendix C
Structure and formulation of constitutive equations for cyclic deformation

C.1 BACKGROUND

Constitutive modelling is a complex subject. The following does not form an introduction to constitutive modelling, nor provide guidance on how to formulate a specific model. It does, however, discuss some of the key terminology and outlines some models which are becoming increasingly popular.

C.2 TYPES OF MODEL

It is possible to define two broad classes of constitutive model: 'Standard' and 'Unified'. Standard models are based on a decomposition of the total strain into elastic, plastic, creep and possibly anelastic contributions. Separate, and usually unrelated, models are formulated to describe the evolution of each strain contribution. These are summed to obtain the total strain. Such models are practical, pragmatic, and often accurate. However, problems arise when interactions occur between the various strain contributions; for example the well known, but little understood, interaction between creep and plasticity. In an attempt to overcome this shortcoming the Unified models were conceived in which creep and plastic strain are essentially the same (they are both related to dislocation motion) and should, therefore, be described by an 'inelastic' strain term. The concept suffers a drawback since creep and plasticity operate on different time-scales. This means that these models require copious computation which makes them difficult to use for practical engineering calculations. They are of use when creep and plasticity occur simultaneously but, in general, components either operate under creep or plasticity conditions; *e.g.* when creep occurs at one end of the cycle and plasticity at the other. Moreover, creep and plasticity differ with respect to stress thresholds. Creep has no clearly definable threshold whereas a substantial elastic range often precedes plastic deformation. Therefore, from an engineering viewpoint, the 'Standard' model is generally preferred.

The remainder of this appendix discusses some aspects of 'Standard' models for plastic (time independent) deformation. Within the scope of this document it is noted that testing at high temperatures (in the creep regime) is not excluded. Under such conditions materials may undergo cyclic creep (rather than cyclic plasticity in the 'true' sense); which is not considered in this appendix.

C.3 NOTES ON 'STANDARD' PLASTICITY MODELS

It is necessary to distinguish between a constitutive model for tracing the history of deformation of a component and the widely used Ramberg–Osgood equation (Eqn (1) [16]). This equation is merely a process equation and describes the cyclic strain range as a function of the cyclic stress range for rate-independent plastic deformation. It is useful but is distinguished from true constitutive equations in that it is not expressed in incremental form. Due to the dissipation of energy during plastic deformation the process is history, or path, dependent. Thus, there will not be a one-to-one correspondence between stress and strain during plastic deformation. This limits the applicability of the Ramberg–Osgood equation to describing particular CSS curves and means that it is not suitable for modelling the response of the material when subjected to complex loading histories.

For this purpose, it is necessary to formulate models in incremental form such that the increment of plastic strain is related to the current magnitude and increment of stress, and perhaps some state variables used to account for certain aspects of past deformation. The accumulated strain is computed by summing the increments of strain. The concept of a yield stress and the need for an incremental formulation are crucial to 'Standard' plasticity models. Each of the models has its own particular strengths and weaknesses which means that no model is universally applicable to all loading conditions and metals.

C.4 MATHEMATICAL CONCEPTS

A yield surface is introduced largely for computational convenience. It is a concept which has a number of associated mathematical 'rules' which include: normality, consistency, loading-unloading criterion, *etc.* A yield surface is also convenient for the analysis of structures (by say, the finite element method) since one can distinguish between regions which are elastic and plastic. Considerably less computational effort is required for elastic calculations than for elasto-plastic calculations. A yield surface is often used when describing the monotonic plastic behaviour.

Plasticity theory requires that the stress point remains on the yield surface as the material is plastically deformed. To achieve this, and to account for changes in the magnitude of the stress (*i.e.* perfect plasticity excluded) as plastic strain increases, the yield surface must change its shape by expanding, contracting, translating or distorting. Metallic materials may exhibit any or all of these features, however, for practical purposes distortion of the yield surface is neglected and only expansion/contraction with or without translation are considered. This leads to the definitions of isotropic and kinematic hardening.

Isotropic hardening assumes that the subsequent yield surface is a uniform expansion of the initial yield surface, and the centre of initial and subsequent yield surfaces are the same. If the yield surface is a circle in stress space then only the radius of this circle can increase during plastic deformation. Isotropic hardening is the simplest to use but it cannot predict the Bauschinger effect. Isotropic softening (*i.e.* contraction of the yield surface) is the opposite of isotropic hardening. Isotropic hardening can be readily modelled by introducing a term into the expression for the yield surface:

$$F(\underline{s}, k) = f(\underline{s}) - k - k_0 = 0 \qquad \text{(C1)}$$

where k represents the isotropic hardening, k_0 is the initial size of the yield surface and $\underline{s}$ is the deviatoric stress. Characters in underlined notation represent tensors. The concept of isotropic hardening is illustrated in Figure C1.

Kinematic hardening assumes that the yield surface translates as a rigid body in stress space during plastic deformation. As a result the shape of the subsequent yield surface remains unchanged. Geometrically the kinematic hardening variable ($\underline{\alpha}$, also known as the back stress) represents the translation of the yield surface. This model can be written:

$$F(\underline{s}, \underline{\boldsymbol{\alpha}}) = f(\underline{s} - \underline{\boldsymbol{\alpha}}) - k_0 = 0 \qquad \text{(C2)}$$

This variable defines the centre of the yield surface in stress space. The concept of kinematic hardening is illustrated in Figure C2.

Again it is noted that neither isotropic nor kinematic hardening is truly representative of actual material behaviour. Nevertheless in some circumstances such as proportional loading, these models can provide satisfactory results.

In general, the yield surface function can be written to include any number of isotropic or kinematic hardening parameters, thereby offering considerable flexibility in representing the response of a material. Test data guide the selection of an appropriate mathematical form, define the relative importance of evolutionary isotropic and kinematic hardening, and are used to identify the parameters of the constitutive model.

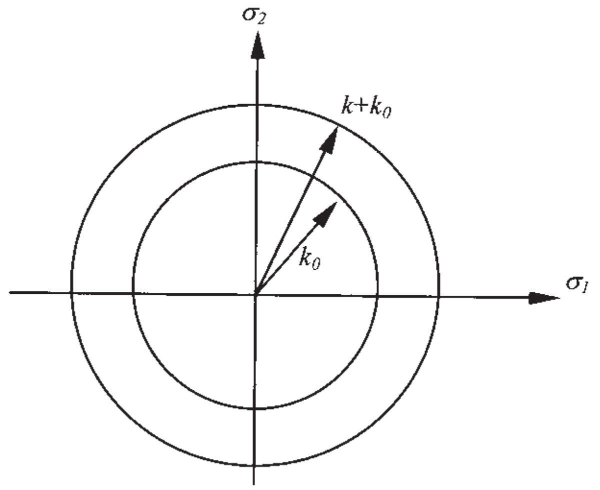

Figure C1 Schematic representation of isotropic hardening.

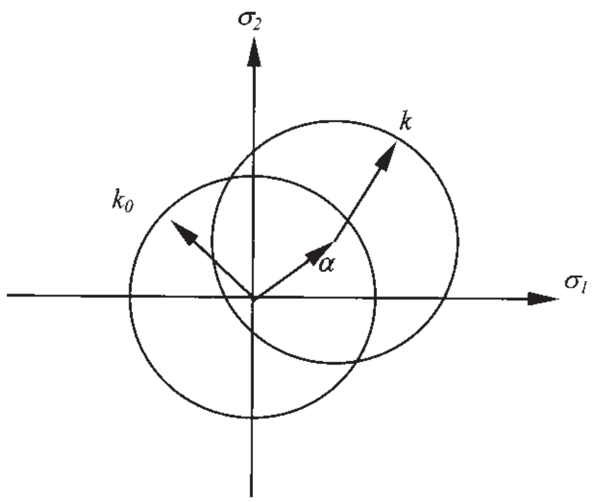

Figure C2 Schematic representation of kinematic hardening.

C.5 SOME PARTICULAR CONSTITUTIVE EQUATIONS

Conventional, associative, plasticity theory enables the increment of plastic strain ($d\varepsilon_p$) to be expressed as

$$d\varepsilon_p = \lambda\ (df/d\sigma) \tag{C3}$$

where λ is some plastic multiplier to be determined. This equation is widely applicable and the difference between various plasticity models is how they describe the evolution of hardening or softening; *i.e.* by isotropic or kinematic hardening.

C.5.1 Linear kinematic hardening

When even the simplest of material responses is investigated it is apparent that isotropic hardening alone cannot describe the behaviour. The most common example of this is the Bauschinger effect. Therefore kinematic hardening is required. The simplest way to effect this is with Prager's linear kinematic rule which introduces proportionality between the back stress (α) and the plastic strain (ε_p):

$$d\alpha = C\ d\varepsilon_p \tag{C4}$$

where C is the 'plastic tangent modulus'. The resulting stress-strain loops have a bi-linear form (Figure C3)

This model is convenient because of its simplicity. It is implemented in several finite element codes. A slightly more advanced version of the model is the ORNL constitutive model [18] developed to describe Type 316 stainless steel and 2.25Cr1Mo steel, among others. The parameters (yield stress, k_0, and tangent modulus, C) are obtained directly from the saturated CSS curve since:

$$\Delta\sigma = 2k_0 + C\Delta\varepsilon_p \tag{C5}$$

The main drawback of the model is the linearity of the CSS curve (the bi-linear hysteresis loops are less of a concern) which means that the true behaviour can only be approximated over a limited set of strain ranges. If this constitutive equation is used to predict the response of a component then the likely strain range must be known so that suitable coefficients can be selected. Two methods are widely accepted for estimating the parameters. Firstly the bi-linear curve is constructed such that it passes through the actual stress-strain curve at the anticipated maximum strain range and at half the maximum strain range. Alternatively the bi-linear curve is constructed such that there is an equal area under the actual stress-strain curve and its bi-linear approximation up to the maximum strain range of interest. The former method, based on a maximum strain range of 1%, is illustrated in Figure C4).

The need for prior knowledge of the maximum strain range means that an estimate has to be made (based on, say, Neuber's method [33]) to determine a preliminary set of parameters. An analysis is then performed to ascertain a more precise estimate of the maximum strain range and, if necessary, the parameters are re-evaluated based on this new estimate.

The predictions of this model are adequate if a single region of the component undergoes a relatively limited amount of cyclic plasticity. If several regions of the component suffer cyclic plasticity then it is unlikely that the bi-linear description will be sufficiently accurate for all regions. Such cases require a more advanced, non-linear, model which accounts for cyclic hardening or cyclic

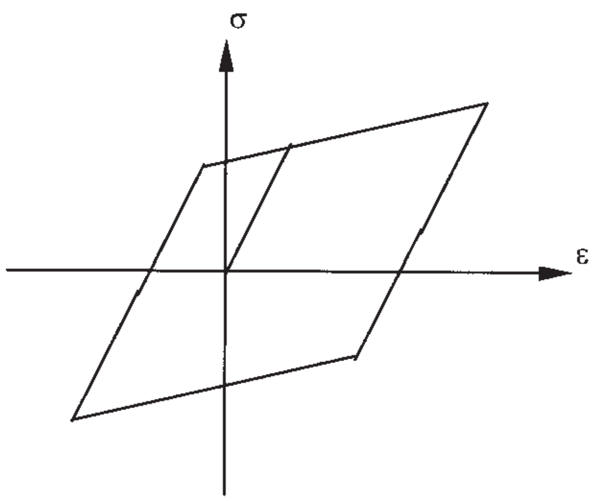

Figure C3 Schematic representation of bi-linear kinematic hardening.

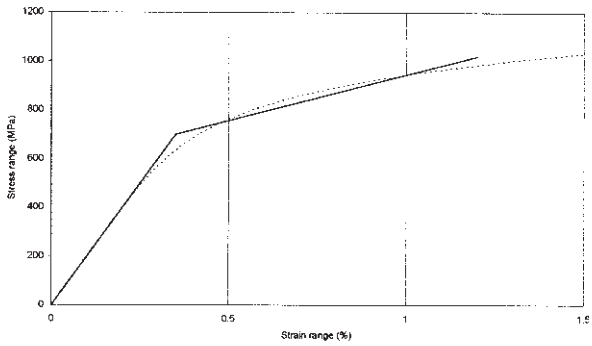

Figure C4 Construction of bi-linear model (solid line) from CSS data (dotted line).

softening (and potentially other aspects of history dependence).

C.5.2 Non-linear hardening

A variety of models have been developed to overcome the limitations of the bi-linear model. The essential features that distinguish these models from one another are the evolution equations for $\boldsymbol{\alpha}$ and k (the kinematic and isotropic variables). These models seek to accurately describe the non-linearity of stress-strain loops and the CSS curve.

The non-linearity is introduced in the following ways:

- In the Mroz model, by defining a field of hardening moduli associated with several surfaces which are originally concentric and which can translate rigidly and expand uniformly.

- In the Dafalias and Popov model, by modelling the continuous variation of hardening modulus based on the concept of two surfaces: the loading surface and the bounding surface.

- In the Armstrong–Frederick model, by direct generalisation of Prager's linear kinematic model:

$$d\underline{\boldsymbol{\alpha}} = C\, d\varepsilon_p - \gamma\, \underline{\boldsymbol{\alpha}}\, |d\varepsilon_p| \qquad \text{(C6)}$$

The first term on the right hand side is identical to the linear kinematic model. The key in this model is the second term on the right hand side, called the recall term, which will affect plastic flow differently for tensile or compressive loading because it depends on $|d\varepsilon_p|$ and is very important in predicting the non-linear CSS loop.

Each of these models has advantages and disadvantages and each must be considered with reference to both the component to be analysed and the characteristics of the material from which it is made. However, the Armstrong-Frederick model has been developed quite extensively by Chaboche and collaborators, and the modified versions are capable of representing many aspects of the material response. The popularity of the modified model has led to its implementation in finite element codes [34].

C.5.3 Chaboche (non-linear kinematic) constitutive model

In its most general form the modified version of the Armstrong–Frederick equation models both isotropic and kinematic hardening (introducing a hardening memory which is not discussed here).

Kinematic variables: $d\underline{\boldsymbol{\alpha}}^m = C^m\, d\varepsilon_p - \gamma^m \underline{\boldsymbol{\alpha}}^m\, |d\varepsilon_p|$

$$\alpha = \sum_m \alpha^m \qquad \text{(C7)}$$

Isotropic variable: $dk = b(Q-k)\, |d\varepsilon_p|$ (C8)

Yield surface: $F = f(\underline{\boldsymbol{s}}-\underline{\boldsymbol{\alpha}}) - k - k_o = 0$ (C9)

A key feature is the introduction of several kinematic variables which are summed to give the complete kinematic effect. This provides considerable flexibility when fitting CSS data and, in principle, the quality of fit is unlimited; although many kinematic variables introduce many parameters, the uniqueness of which becomes increasingly uncertain. In practice it is common to use two kinematic variables with one providing a good description of the non-linearity at low strain ranges and the other at high strain ranges. The use of an accompanying isotropic variable is optional and depends on the hardening characteristics of the material.

The principal drawback of the model is its tendency to predict ratchetting under a non-symmetric stress cycle. Under strain control any initial mean stress will relax to zero during the first few cycles. Some materials exhibit varying degrees of ratchetting and therefore the characteristics of the material must be ascertained prior to the use of this model. The predicted ratchetting effect is a direct result of the formulation of the recall term which contains the back stress. Without the recall term no ratchetting or relaxation occurs (as for the linear kinematic model). The superposition of several kinematic variables can lessen this ratchet effect particularly if one of them is linear. However, if non-ratchetting is a prerequisite then it may be more appropriate to use another type of model.

The non-linear kinematic term(s) essentially describe the non-linearity in the hysteresis loops at stabilised cyclic conditions. Gradual cyclic hardening or softening of an isotropic nature (change in cyclic yield stress) can be readily described with the isotropic variable. If, however, the hardening is kinematic (change in plastic tangent modulus), then it is necessary to make the parameters C and γ in the expression for kinematic hardening functions of plastic work, plastic path length or some other accumulated variable.

Appendix D
Exploitation of data (worked example for cyclically hardening material)

D.1 INTRODUCTION

The structural analysis of components operating at elevated temperatures often requires a description of the cyclic deformation behaviour of the constituent materials. Despite considerable effort in developing models, which give a full description of the hardening behaviour of materials at elevated temperature, these have not been widely implemented. Instead, simplified procedures are often employed which utilise the simple representation of the peak values of stress and strain based on the Ramberg–Osgood equation, as discussed earlier.

As noted in Section 7.2, the saturated CSS properties of many materials are widely available in the published literature. By comparison, the evaluation of the coefficients for evolutionary models require more detailed test records which are rarely available. When such results have been reported they have tended to refer to single casts of materials and have usually been generated to test constitutive models rather than provide mean and bounding materials properties.

This appendix considers how the published data can be best utilised. Analyses of data for unstabilised austenitic stainless steels AISI Type 304 are presented for illustrative purposes.

D.2 ANALYSIS OF STABILISED CSS DATA

Saturated values of $\Delta\sigma$ and $\Delta\varepsilon_t$ for Type 304 stainless steels at a range of temperatures have been analysed according to the method detailed in Section 7.2. The resulting values of the Ramberg-Osgood coefficients, together with upper and lower bound values of the term *A* are summarised in Table D1.

D.3 EFFECT OF TEMPERATURE

It is not possible to anticipate the relative strengths of a material at different temperatures by simple inspection of the tabulated coefficients *A* and β. Consequently, for the case of the Type 304 stainless steel, the cyclic stress ranges ($\Delta\sigma$) corresponding to total plastic strain ranges between 0.2% and 1% have been calculated and are plotted as a function of temperature in Figure D1. There is a discernible trend, which is similar to the temperature dependence of the tensile strength, R_m, for Type 304 steel.

Table D1 Ramberg-Osgood coefficients for Type 304 stainless steel

		A (MPa)		
Temperature (°C)	β	Mean	Upper 95%	Lower 95%
20	0.361	3416	4646	2470
400	0.483	4977	5854	4081
450	0.463	5589	6958	4220
500	0.350	3332	4091	2574
540 to 570	0.242	1875	2451	1300
600	0.227	1470	1805	1136
650	0.208	1164	1587	741

To remove some of the variability arising from the variation in the number of data and casts tested at each temperature, the data shown in Figure D1 were fitted to a third order polynomial function to give a best estimate of the stress range as a function of temperature. It must be emphasised that the relationships between cyclic stress range and temperature have no physical significance and were chosen simply to provide a smooth description of the overall behaviour. Furthermore, it is well known that polynomial functions can give extreme values outside the range of the data and **therefore this approach must not be used to predict behaviour outside the range of temperatures tested**. Revised values of *A* and β have been derived by fitting these best estimate values of stress range to Eqn. 3 and these are reported in Table D2. As a consequence of this revised analysis it is not possible to derive formal confidence limits. On the basis of the typical range of upper and lower 95% confidence values for *A* given in Table D1, it is proposed that confidence limits corresponding to ±25% of the mean values of *A* should be used for assessment purposes.

D.4 ANALYSIS OF EVOLUTIONARY CSS DATA

In some circumstances the number of cycles experienced by a component in service may be small and the constituent materials will not reach fully saturated conditions. It may then be necessary to take advantage of the lower, but continually changing, cyclic strength of the material. State variable models of plasticity use isotropic and kine-

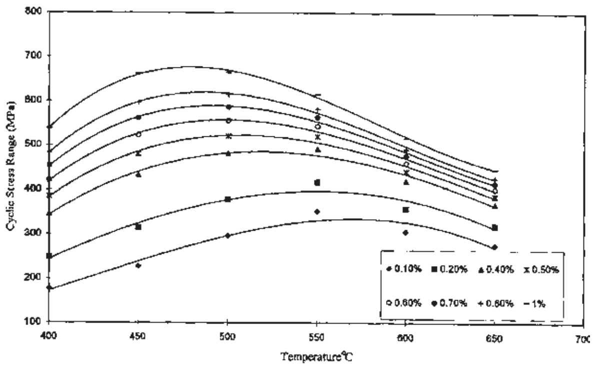

Figure D1 Cyclic stress ranges as a function of temperature.

Table D2 Ramberg–Osgood coefficients for Type 304 steel based on a polynomial fit across the temperature range 400–650°C

Temperature (°C)	β	*A* (MPa)
400	0.493	5215
450	0.441	5034
500	0.354	3417
550	0.264	2054
600	0.199	1303
650	0.215	1199

matic hardening parameters, which are designed to provide a full description of the cycle by cycle deformation behaviour. Goodall *et al.* [35] proposed that the peak cyclic stress value at any strain range could be described by a relationship of the form:

$$\frac{\sigma - \sigma_o}{\sigma_\infty - \sigma_o} = 1 - \exp(b \cdot P_p) \qquad \text{(D1)}$$

where σ is the peak cyclic stress, σ_0 and σ_∞ are the initial and saturated peak values of stress respectively and P_p is the accumulated plastic strain. For Type 316 stainless steel at 600°C and a total strain range of 0.5%, Goodall *et al.* found *b* to have the value 0.0385 (for P_p in %) [35].

A series of tests on Type 304 steel at 650°C has been analysed according to Eqn (D1). Initial results were rather inconsistent and the quality of fit to Eqn (D1) varied with strain range. This was shown to be due, in part, to difficulties associated with assigning values of the limiting stresses, in particular the saturated value, although the steady-state values quoted in Table D2 can be used as a first approximation. By treating σ_0 and σ_∞ as fitted parameters a more consistent set of results was obtained and these are presented in Table D3.

An alternative, though similar, relationship due to Skallerud and Larsen [36] replaces the term $(b.P_p)$ in Eqn (D1) by:

$$\zeta \frac{N}{N_{sat} - N}$$

where N is the number of cycles, N_{sat} is the number of cycles to saturation and ζ is a constant This relationship appears to be empirical. However, if the relationship is approximated to N/N_{sat} and N_{sat} is treated as a fitted parameter, it found that N_{sat} is related to the plastic strain range. It is worth noting that for cyclically hardening materials saturation is often observed to occur at about 5 or 10% of the cycles to failure. In LCF, the number of cycles to failure, N_f, is related to the plastic strain range by the Coffin-Manson relationship:

$$(N_f)^{\phi} \cdot \Delta\varepsilon_p = C_3 \qquad \text{(D2)}$$

where C_3 is a constant.

Table D3 Results of an analysis of the evolutionary cyclic hardening of Type 304 steel

	Total strain range (%)	Strain rate (%/s)	Initial stress (MPa)	Saturated stress (MPa)	b ($\%^{-1}$)
1	0.3	0.025	80.3	128.7	0.011
	0.6	0.025	94.2	175.0	0.019
	0.9	0.025	95.4	185.3	0.024
	1.2	0.025	97.6	190.6	0.023
	1.5	0.025	101.4	210.8	0.069
	2.0	0.025	97	223.3	0.053
	0.6	0.006	86.0	155.3	0.032
	2.0	0.002	97.6	198.5	0.096
2	0.3	0.025	75.7	110.2	0.011
	0.6	0.025	88.0	156.8	0.027
	0.9	0.025	90.8	179.4	0.023
	1.2	0.025	102.2	194.3	0.023
	2.0	0.025	107.2	232.1	0.041

1* Refers to tests on as received material.
2* Refers to tests on pre-aged material.

A plot of the plastic strain range versus the number of cycles to saturate, shown in Figure D2, gives a value of $\phi = 0.58$ and a constant of 0.067. This is in good agreement with the observation of Manson [37] that the exponent ϕ is universally 0.6. The relationship also corresponds with fatigue endurance data for austenitic steel at 650°C but reduced by a factor of 20, confirming the empirical observation that saturation of cyclic hardening occurs at about 5% of fatigue life.

This modified analysis leads to a simple relationship for the exponent of the plastic strain ranges in Eqn (D3). Since $P_p = 2.N.\Delta\varepsilon_p$ and $N_{sat} \approx N_f/20 = (\Delta\varepsilon_p/C_3)^{-1/\phi}/20$, it can be shown that:

$$b = 10 \cdot (C_3)^{1/\phi} . (\Delta\varepsilon_p)^{(1/\phi)-1} \qquad \text{(D3)}$$

Plotting the cyclic hardening properties of Type 304 steel gives a value of the exponent in Eqn (D3) of 0.83 which corresponds to a value of ϕ of 0.54. This may be compared with a value of 0.58 for the LCF endurance of austenitic steels.

D.5 IN THE ABSENCE OF CYCLIC DATA

In the absence of detailed cyclic deformation data, the approach outlined above can be developed to give an approximate description of cyclic behaviour based on more generally available data; in particular monotonic tensile data and fatigue endurance data. In the event that even the latter information is unavailable then it too can be estimated using the Manson–Coffin equation and the observation of "Universal slopes". Thus:

$$\Delta\varepsilon_t = \left(\frac{B_2}{E}\right) \cdot N_f^{-1/\Gamma_2} + B_1 \cdot N_f^{-1/\Gamma_1} \qquad \text{(D4)}$$

where $B_1 = (-\ln(1-Z))^{0.6}$, $1/\Gamma_1 = 0.6$, $B_2 = 3.5.R_m$, and $\Gamma_2 = 8.33$. It can be further shown that:

$$\beta = \frac{\Gamma_1}{\Gamma_2} \text{ and } A = \frac{B_2}{B_1^{\beta}}$$

At a given strain range, the initial stress is derived from the monotonic tensile properties. The number of cycles to failure at the same strain range can be estimated using the Manson–Coffin equation (Eqn (D2)). The saturated

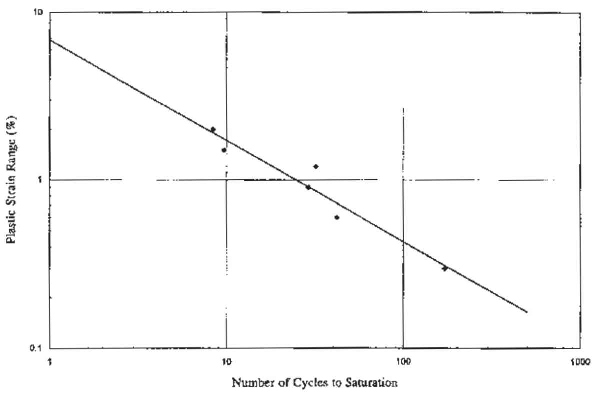

Figure D2 Relationship between number of cycles to reach saturation, N_{sat}, and plastic strain range.

stress range can be estimated using the derived values of A and β, as shown above. For the case of Type 304 stainless steel examined here, the cyclic hardening at 650°C and total strain ranges of 0.6% and 1.5% has been determined using published tensile data and an estimated mean R_m of 280 MPa and $Z = 50\%$. These properties yield values of $\beta = 0.2$ and $A = 1097$ MPa which can be compared with the experimentally determined values given in Table D2 of 0.215 and 1199 MPa respectively.

Thus, for materials which harden according to an exponential relationship and which saturate at about $N_f/20$ cycles, it is shown that the cyclic evolution of stress is given by

$$\frac{\Delta\sigma - \Delta\sigma_o}{\Delta\sigma_\infty - \Delta\sigma_o} = 1 - \exp\left(\frac{-2 \cdot N}{(N_f / 20) - N}\right) \qquad \text{(D5)}$$

A summary of the results obtained is given in Table D4 together with the cast-specific values determined experimentally.

The cyclic changes in stress range for a total strain range of 0.6% and 1.5% are shown as a function of N/N_{sat} in Figure D3. It can be seen that the greatest deviation between the calculated and observed behaviour is in the initial few cycles and is due to a significant difference between the published mean tensile data and the cast-specific tensile data. It is clear that a simple tensile test on cast-specific material significantly improves the predicted behaviour.

D.6 FITTING TO CHABOCHE CONSTITUTIVE EQUATIONS

The methodology outlined in Section 7.3.5 has been used to determine the parameters of the Chaboche constitutive model. For consistency with Section 7.3.5 two non-linear kinematic hardening variables have been used and, since only a description of the stabilised CSS response is being sought, isotropic hardening has been neglected.

The equations have been fitted to the stabilised CSS data given in Table D3. The stabilised plastic strain range for each test was determined from Eqn (3) by using an elastic modulus of 130 GPa. Non-linear regression, treating the plastic strain range as the independent variable, was then performed to determine the parameters of Eqn (9). This procedure is similar to that employed to fit the Ramberg-Osgood equation (Eqn (4)). Because of the large number of parameters in Eqn (9), it is necessary to demonstrate that their values are unique and that the global minimum was attained in the optimisation procedure. This is most simply effected by adjusting the initial guess for each parameter, repeating the regression and checking that the values of the optimal parameters are essentially unchanged. The values of the parameters are summarised in Table D5. Figure D4 shows that the parameters provide an excellent fit to the CSS range data.

At this point the description of the CSS response, defined by Eqn (9), could be used for simplified component assessments in the same way as the Ramberg–Osgood equation. The principal advantage of the Chaboche (or similar) model is that it can be used to obtain predictions of the response of structures subject to complex loads; this is, however, beyond the scope of this document. Nevertheless, it is straightforward to obtain predictions of the stabilised hysteresis loops under uniaxial cycling. Figure D5 compares a selection of the hysteresis loops for Type 304 steel with the predictions from the Chaboche model. It can be seen that the equations give an excellent representation of the peak values of the hysteresis loops but do not provide a good representation of the intermediate deformation response. This apparent discrepancy is not of great concern if the model is to be used to predict the strain ranges in a structure under cyclic loads. However,

Table D4 A Summary of the cyclic properties of Type 304 steel derived from tensile data

Property	Strain range = 0.6%		Strain range =1.5%	
	Calculated	Observed	Calculated	Observed
$\Delta\sigma_1$	240 MPa[a]	175 MPa	298 MPa[a]	213 MPa
$\Delta\sigma_\infty$	369 MPa	350 MPa	468 MPa	446 MPa
N_f	3000	2700	800	N.A.
N_{sat}	150	170	40	40

[a]Data from N-47 [38]

Table D5 Parameters for the Chaboche constitutive equation to represent Type 304 steel

Parameter	Value
k_0 (MPa)	91.1
C_1 (MPa)	42235.9
γ_1 (–)	757.7
C_2 (MPa)	12477.1
γ_2 (–)	147.9

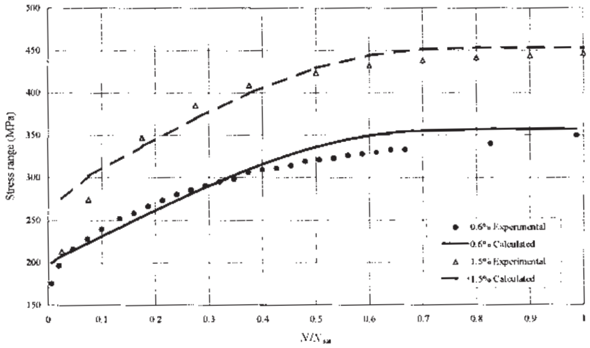

Figure D3 Observed and calculated stress ranges as a function of cycle number for Type 304 steel.

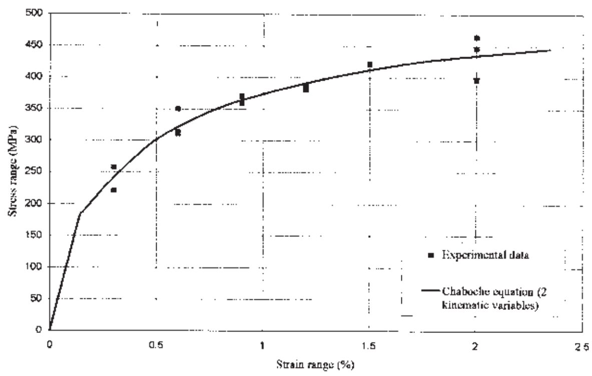

Figure D4 Cyclic stress-strain curve for Type 304 steel (data points taken from Table D3).

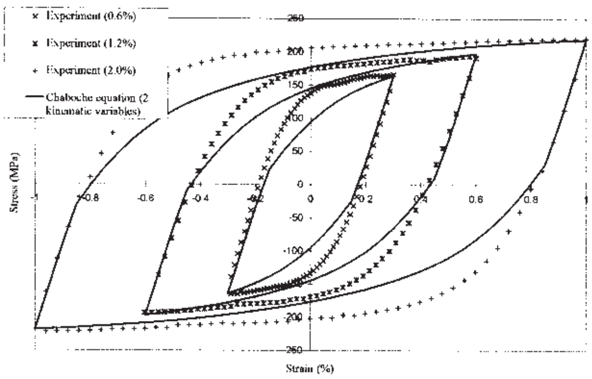

Figure D5 Comparison of experimental and predicted stabilised hysteresis loops for Type 304 steel.

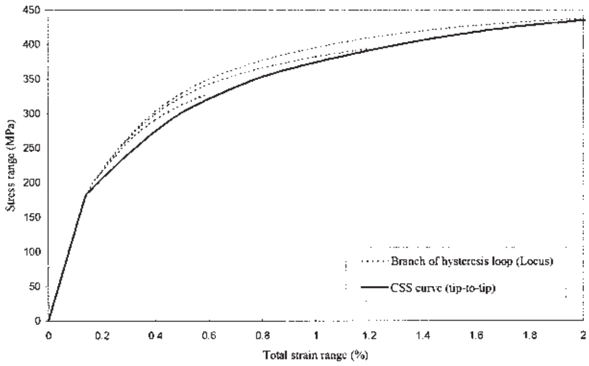

Figure D6 Comparison of CSS curve and individual hysteresis loops to demonstrate the 'non Masing' nature of the Chaboche model.

the predictions from the model would be questionable if, for example, a creep dwell were to occur at an intermediate point during a cycle (stress at the start of the dwell would be underestimated). In this regard it is interesting to compare the CSS curve (tip-to-tip values) with a branch of a predicted hysteresis loop. Such a comparison is shown in Figure D6 which clearly illustrates that the Chaboche model is of 'non-Masing' type. For a dwell part-way through a cycle, it is apparent that the model would, in fact, give a better prediction of the stress at the start of the dwell than the original CSS curve.

The prediction of the hysteresis loops could be improved further by restricting the strain ranges used in the original fit to determine the parameters (if the data points at a strain range of 0.3% are omitted then the apparent cyclic yield stress increases and the description of the hysteresis loops is improved slightly), or by fitting the Chaboche equations directly to a set of hysteresis loops. These and other fitting strategies are discussed in references such as [7] and [21].

Appendix E
Exploitation of data (example for cyclically softening material)

When stress-strain data are available both for the steady state, and also for the first cycle, then it is possible to predict evolutionary behaviour by interpolation of the relevant constants, as shown in Appendix D. The method described here is more general since it interpolates actual values of A and β, thus circumventing the need for separate constants at individual total strain ranges. (*NOTE: The method may also be applied to cyclically hardening materials.)*

E.1 DETERMINATION OF FIRST CYCLE DATA

It has been shown for many alloys at elevated temperature [15,31] that during evolutionary softening or hardening, the intermediate values of A and β (denoted by the suffix i, where i corresponds to cycle number) appear to be coupled so that:

$$\frac{\beta_i}{A_i} \approx 1\times10^{-4} \qquad \text{(E1)}$$

expressed in units of MPa^{-1}. Thus for softening materials the parameter A_i decreases with evolution. The converse applies for cyclic hardening [15,31]. The term β_i changes to suit, according to Eqn (E1) but as discussed in Section 7.2 the largest influence is felt via the parameter A.

It thus follows that parameter values A_o, β_o are required for the first cycle of loading. Strictly, this refers to 1¼ cycle data since large differences can occur between the ¼ cycle (monotonic loading) stage and the completion of the first hysteresis loop, which will not be closed if the rate of hardening or softening is high.

For a given range of total strain, many investigators provide 'families' of the total stress range versus cycles envelope, see for example Figures 4 and 5 of the main text. An estimate of the values of A_o and β_o is obtained as follows:

- For each curve at cycle 1, note the corresponding value of $\Delta\sigma$.
- Note the corresponding value of $\Delta\varepsilon_t$ and determine a likely value for the elastic modulus, E.
- Calculate the plastic strain range $\Delta\varepsilon_p$ from Eqn (3) (first part), see also Eqn (B6).
- Determine A_o and β_o by least mean squares curve fit to Eqn (4).

E.2 INTERPOLATION METHOD

Using Eqns (D1) and (D5) as a guide, Eqn (E1) is satisfied to a very good degree of approximation at any stage by:

$$A_i = A_o + (A-A_o)[1-\exp(-kN/N_s)] \quad \text{(E2)}$$

and:

$$\beta_i = \beta_o + (\beta-\beta_o)[1-\exp(-kN/N_s)] \quad \text{(E3)}$$

It is re-emphasised that the terms A and β still refer to steady-state values, determined by the methods described in Section 6.2 and Section 7.2. Values for k may be determined by inspection. As a guide, $k = 8$ for softening materials and $k = 12$ for hardening materials, *i.e.* the latter appear to approach 'saturation' more quickly. It appears that there is very little sensitivity of k to strain range. The term is not related to that used in Section C.5.3.

E.3 EXAMPLE FOR A SOFTENING MATERIAL

Nagesha *et al*. [39] have published softening curves for an advanced ferritic steel. Some of their data for 550°C at a reversed strain rate of 3×10^{-3}/s are plotted in Figure E1 for extreme total strain ranges of 2.0% and 0.5% respectively. From tabulated data in the paper, the following steady state values are quoted (separate specimen tests, duly converted from Eqn (5)):

$$A = 922 \text{ MPa} \qquad \beta = 0.120$$

Using the method given in Section E.1 and using the 1st cycle data of Figure E1 together with intermediate total strain range data at 0.8% and 1.2% (not shown) the following values were established:

$$A_o = 1092 \text{ MPa} \qquad \beta_o = 0.108$$

From Figure E1 the 'saturation' cycles, N_s, may be taken as 170 and 2000 at the upper and lower total strain ranges of 2.0% and 0.5% respectively. By means of Eqns (E2) and (E3), values of A_o and β_o were established for several values of cycles up to the respective maxima N_s. (*NOTE: For softening materials, the value of* N_s *may be taken as half life* $N_f/2$. *For hardening materials,* N_s *occurs much sooner, typically at* $N_f/20$, *see Section D.5*.)

Cyclic stress-strain curves may thus be predicted for any value of N during evolution from Eqn (3), now expressed as:

$$\Delta\varepsilon_t = \frac{\Delta\sigma}{E} + \left(\frac{\Delta\sigma}{A_i}\right)^{1/\beta_i} \quad \text{(E4)}$$

These curves may be generated as desired, *e.g.* 10th cycle, 25th cycle (absolute) or at N/N_s (relative) for given strain ranges.

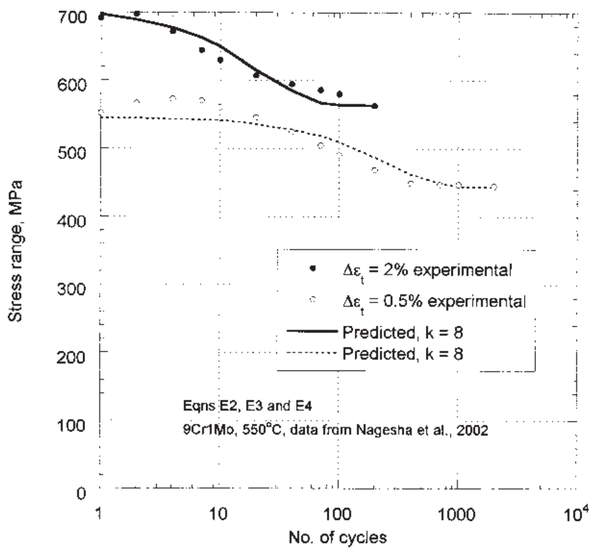

Figure E1 Prediction of cyclic softening in modified 9Cr1Mo steel at 550°C.

For the ferritic steel, Eqn (E4) was solved numerically for several intermediate cycles at 2.0% and 0.5% strain range respectively. The results plotted in Figure E1 give a very reasonable description of the material response. The brief period of hardening (up to 10 cycles) reported for this steel [39] seems to be typical of this material type, and cannot be predicted via Eqns (E2) and (E3).

E.4 IN THE ABSENCE OF FIRST CYCLE DATA

If first cycle data cannot be determined from the method described above, then an approximation may be effected via Eqns (5) and (B3) in the form:

$$A_o = A_m(2)^{1-\beta_o} \quad \text{(E5)}$$

where A_o is the full range value arising in turn from A_m and β_o, determined from the monotonic curve.

Alternatively, if ε_{UTS} is the engineering strain at the UTS, R_m, then another approximation is given by [40]:

$$\beta_o = \ln(1 + \varepsilon_{UTS}) \quad \text{(E6a)}$$

and:

$$A_o = \frac{R_m(1+\varepsilon_{UTS})}{\beta_o^{\beta_o}}(2)^{1-\beta_o} \quad \text{(E6b)}$$